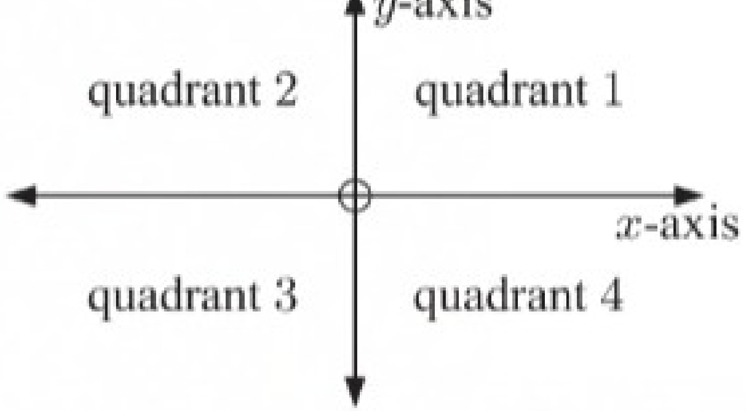
The **number plane** consists of two perpendicular axes which intersect at the **origin**, O.

The ***x*-axis** is horizontal and the ***y*-axis** is vertical.

The axes divide the number plane into four **quadrants**.



The number plane is also known as either the **2-dimensional plane**, or the **Cartesian plane** after **René Descartes**.

The position of any point in the number plane can be specified in terms of an **ordered pair** of numbers (x, y), where:

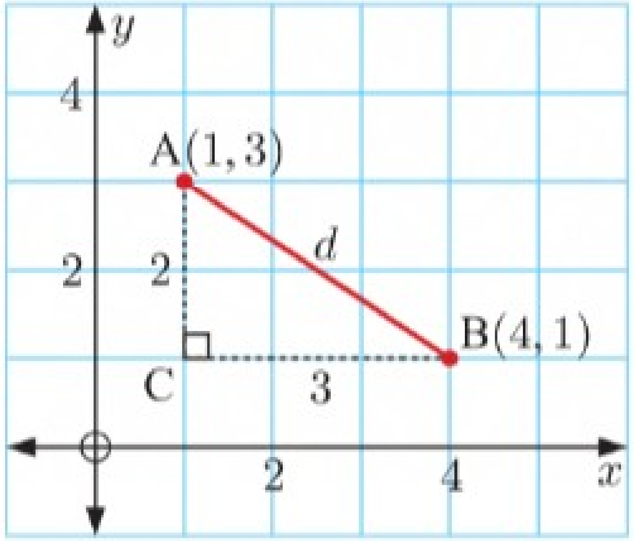
- x is the **horizontal step** from O, and is the x-coordinate of the point
- y is the **vertical step** from O, and is the y-coordinate of the point.

DEMO

A THE DISTANCE BETWEEN TWO POINTS

A THE DISTANCE BETWEEN TWO POINTS

Suppose we want to find the distance d between the points A(1, 3) and B(4, 1).



By drawing line segments [AC] and [BC] along the grid lines, we form a right angled triangle with hypotenuse [AB].

$$\therefore\ d^2 = 2^2 + 3^2 \quad \{\text{Pythagoras}\}$$
$$\therefore\ d^2 = 13$$
$$\therefore\ d = \sqrt{13} \qquad \{\text{as } d > 0\}$$

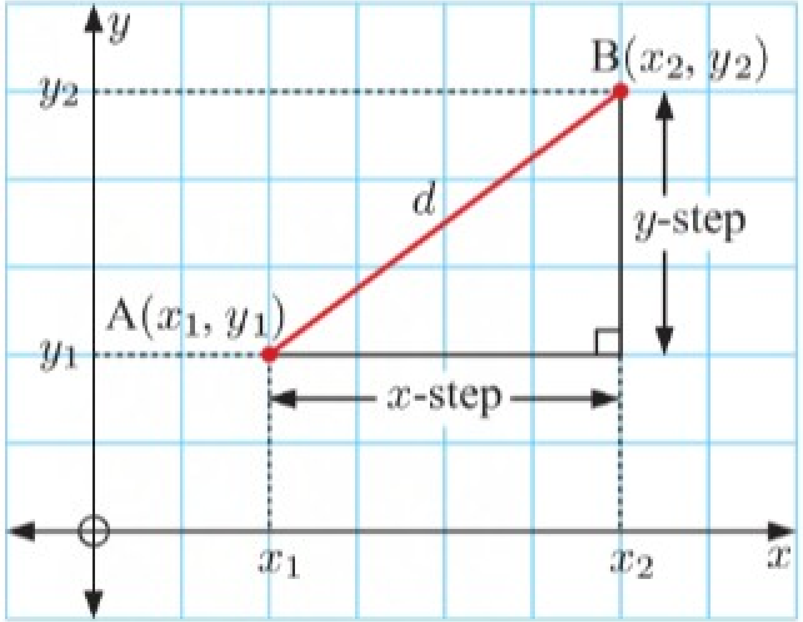
So, the distance between A and B is $\sqrt{13}$ units.

While this approach is effective, it is time-consuming because a diagram is needed.

To make the process quicker, we can develop a formula.

To go from $A(x_1, y_1)$ to $B(x_2, y_2)$, we find the

$$\begin{aligned} x\text{-step} &= x_2 - x_1 \\ \text{and} \quad y\text{-step} &= y_2 - y_1. \end{aligned}$$



Using Pythagoras' theorem,

$$\begin{aligned} (AB)^2 &= (x\text{-step})^2 + (y\text{-step})^2 \\ \therefore \quad AB &= \sqrt{(x\text{-step})^2 + (y\text{-step})^2} \\ \therefore \quad d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \end{aligned}$$

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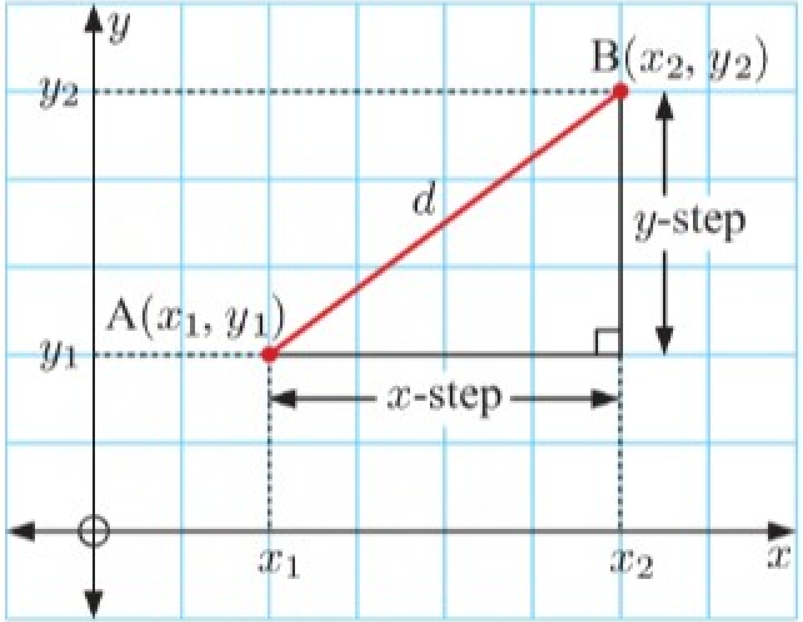


Using Pythagoras' theorem,

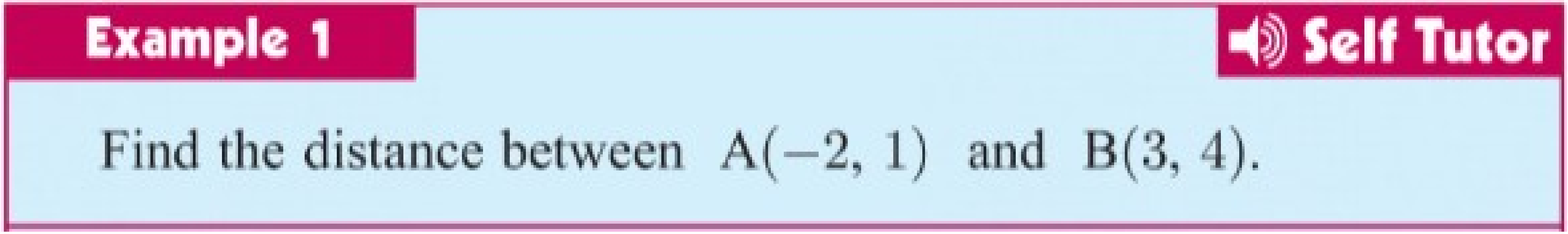
$$(AB)^2 = (x\text{-step})^2 + (y\text{-step})^2$$

$$\therefore \quad AB = \sqrt{(x\text{-step})^2 + (y\text{-step})^2}$$

$$\therefore \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by

$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}\,.$$



Example 1

Self Tutor

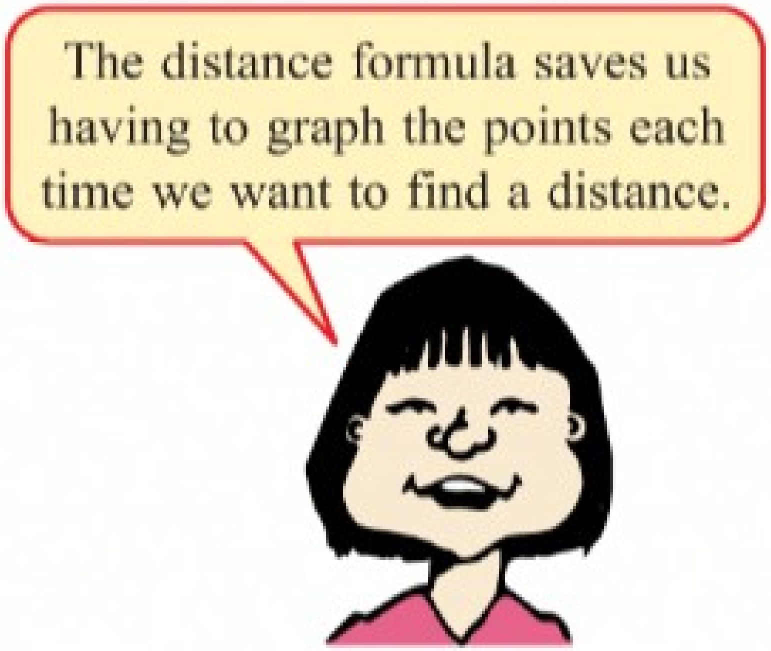
Find the distance between A$(-2, 1)$ and B$(3, 4)$.

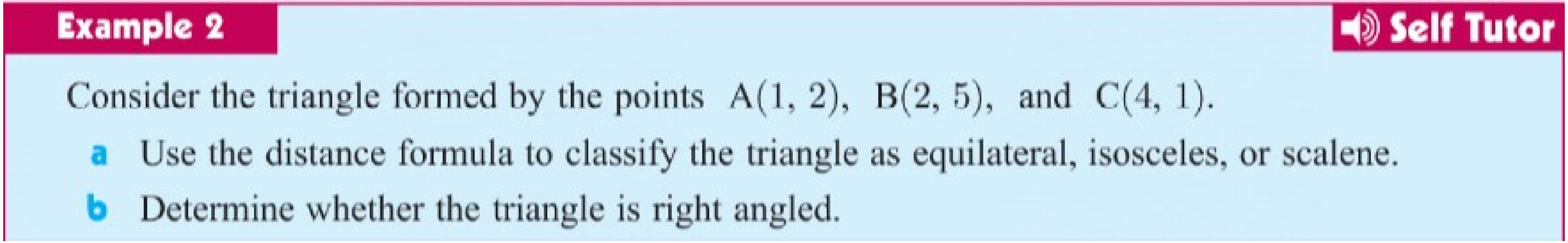
Example 1 Self Tutor

Find the distance between A(−2, 1) and B(3, 4).

$A(\underset{x_1}{\underset{\uparrow}{-2}}, \underset{y_1}{\underset{\uparrow}{1}})$ $\quad B(\underset{x_2}{\underset{\uparrow}{3}}, \underset{y_2}{\underset{\uparrow}{4}})$

$$\begin{aligned} AB &= \sqrt{(3 - -2)^2 + (4 - 1)^2} \\ &= \sqrt{5^2 + 3^2} \\ &= \sqrt{25 + 9} \\ &= \sqrt{34} \text{ units} \end{aligned}$$


The distance formula saves us having to graph the points each time we want to find a distance.



Example 2

Self Tutor

Consider the triangle formed by the points A(1, 2), B(2, 5), and C(4, 1).

a Use the distance formula to classify the triangle as equilateral, isosceles, or scalene.

b Determine whether the triangle is right angled.

Example 2

Self Tutor

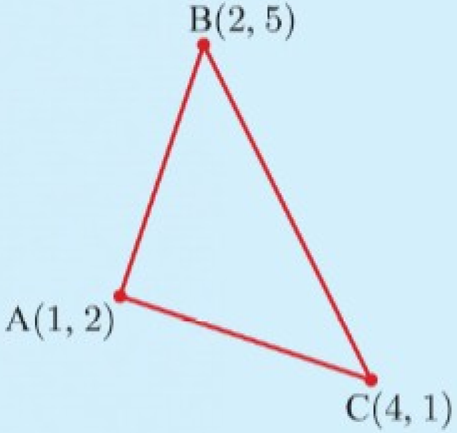
Consider the triangle formed by the points A(1, 2), B(2, 5), and C(4, 1).

a Use the distance formula to classify the triangle as equilateral, isosceles, or scalene.

b Determine whether the triangle is right angled.

a $AB = \sqrt{(2-1)^2 + (5-2)^2}$
$= \sqrt{1^2 + 3^2}$
$= \sqrt{10}$ units

$AC = \sqrt{(4-1)^2 + (1-2)^2}$
$= \sqrt{3^2 + (-1)^2}$
$= \sqrt{10}$ units



$BC = \sqrt{(4-2)^2 + (1-5)^2}$
$= \sqrt{2^2 + (-4)^2}$
$= \sqrt{20}$ units

Since AB = AC, the triangle is isosceles.

b The shortest sides are [AB] and [AC].

Now $AB^2 + AC^2 = 10 + 10$
$= 20$
$= BC^2$

Using the converse of Pythagoras' theorem, the triangle is right angled. The right angle is at A, opposite the longest side.

EXERCISE 6A

1 Find the distance between:

a A(3, 1) and B(5, 3)

b C(−1, 2) and D(6, 2)

c O(0, 0) and P(−2, 4)

d E(8, 0) and F(2, −3)

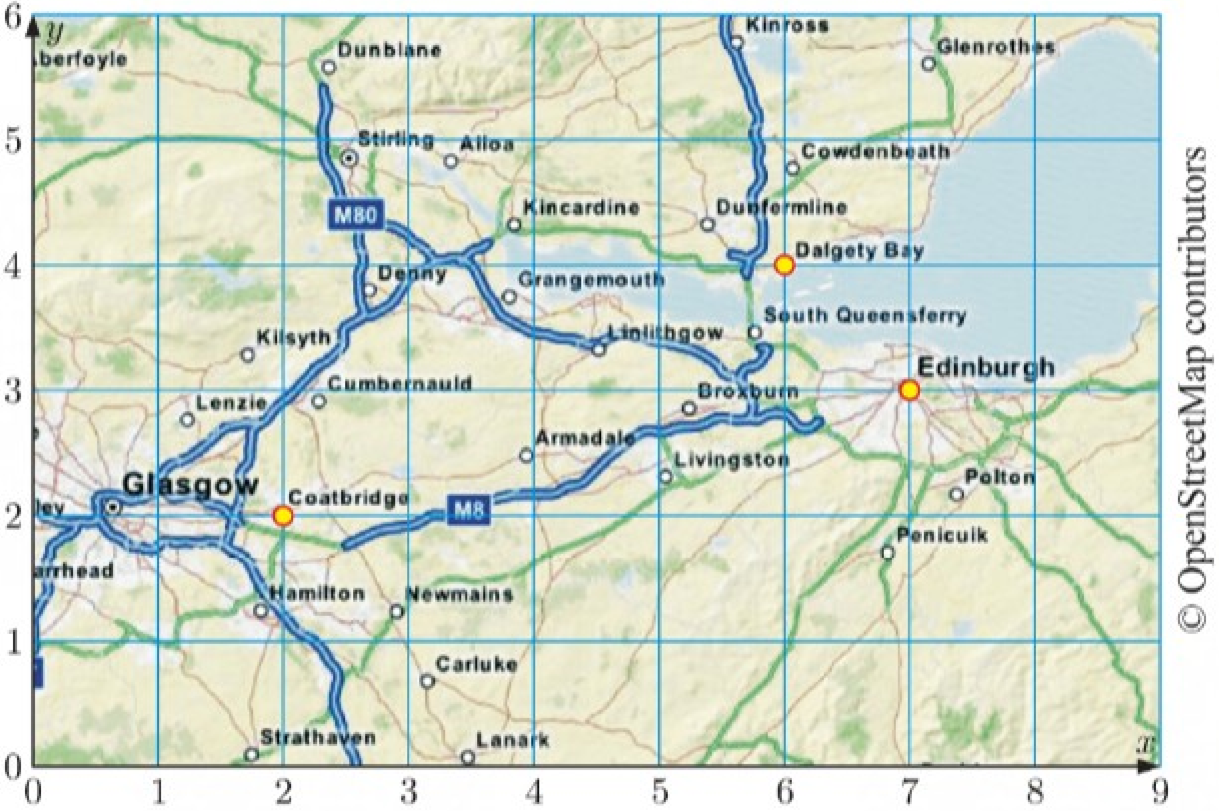
e G(0, −2) and H(0, 5)

f I(2, 0) and J(0, −1)

g R(1, 2) and S(−2, 3)

h W(1, −1) and Z($\frac{1}{2}$, −2).

2 In the map below, the grid lines are 10 km apart.



© OpenStreetMap contributors

Find the direct distance between:

a Dalgety Bay and Edinburgh **b** Coatbridge and Dalgety Bay **c** Coatbridge and Edinburgh.

3 Use the distance formula to classify triangle ABC as either equilateral, isosceles, or scalene:

a $A(3, -1)$, $B(1, 8)$, $C(-6, 1)$

b $A(1, 0)$, $B(3, 1)$, $C(4, 5)$

c $A(-1, 0)$, $B(2, -2)$, $C(4, 1)$

d $A(\sqrt{2}, 0)$, $B(-\sqrt{2}, 0)$, $C(0, -\sqrt{5})$

e $A(\sqrt{3}, 1)$, $B(-\sqrt{3}, 1)$, $C(0, -2)$

f $A(a, b)$, $B(-a, b)$, $C(0, 2)$

4 Determine whether the following triangles are right angled. If there is a right angle, state the vertex where it occurs.

a $A(-2, -1)$, $B(3, -1)$, $C(3, 3)$

b $A(-1, 2)$, $B(4, 1)$, $C(4, -5)$

c $A(1, -2)$, $B(3, 0)$, $C(-3, 2)$

d $A(3, -4)$, $B(-2, -5)$, $C(-1, 1)$

Example 3 **Self Tutor**

Find b given that $A(3, -2)$ and $B(b, 1)$ are $\sqrt{13}$ units apart.
Explain your result using a diagram.

Example 3

Self Tutor

Find b given that A$(3, -2)$ and B$(b, 1)$ are $\sqrt{13}$ units apart.
Explain your result using a diagram.

From A to B, x-step $= b - 3$

y-step $= 1 - -2 = 3$

$$\therefore \quad \sqrt{(b-3)^2 + 3^2} = \sqrt{13}$$

$$\therefore \quad (b-3)^2 + 9 = 13 \quad \{\text{squaring both sides}\}$$

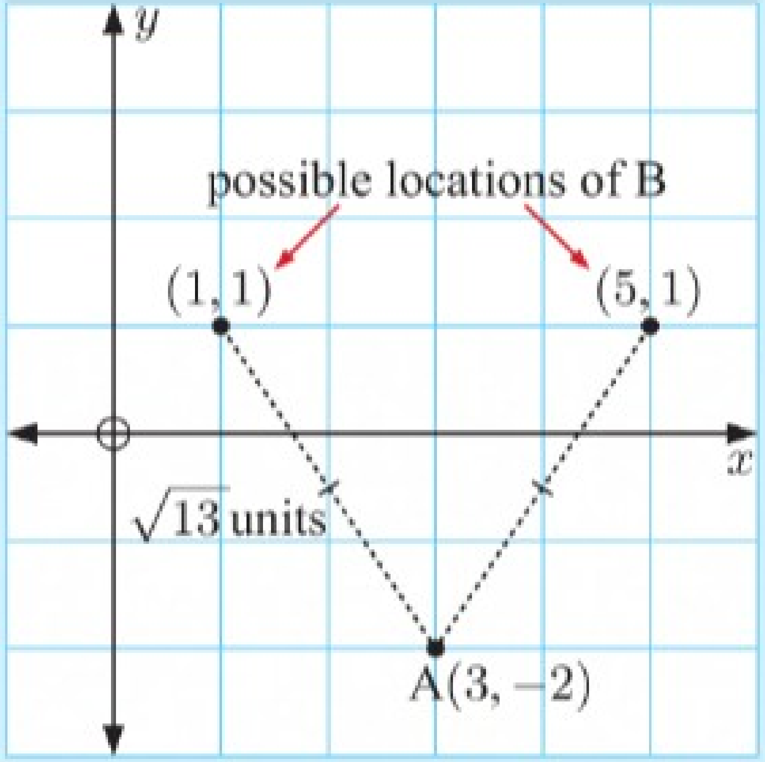
$$\therefore \quad (b-3)^2 = 4$$

$$\therefore \quad b - 3 = \pm 2$$

$$\therefore \quad b = 3 \pm 2$$

$$\therefore \quad b = 5 \text{ or } 1$$

Point B could be at two possible locations:
$(5, 1)$ or $(1, 1)$.



5 For each of the cases below, find a and explain the result using a diagram:

a P$(2, 3)$ and Q$(a, -1)$ are 4 units apart

b P$(-1, 1)$ and Q$(a, -2)$ are 5 units apart

c X(a, a) is $\sqrt{8}$ units from the origin

d A$(0, a)$ is equidistant from P$(3, -3)$ and Q$(-2, 2)$.

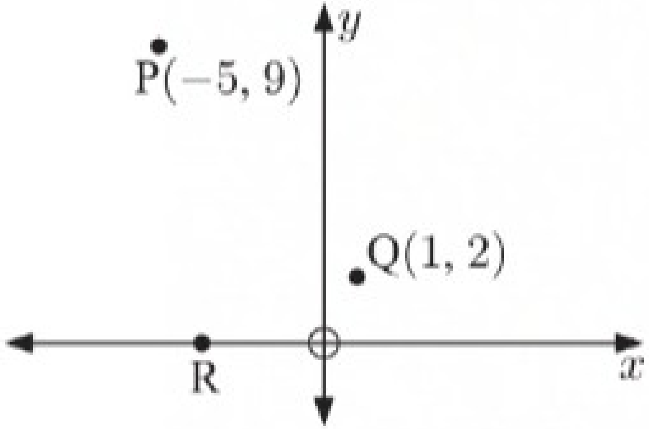
6 **a** Find the relationship between x and y if the point P(x, y) is always:

i 3 units from O$(0, 0)$

ii 2 units from A$(1, 3)$.

b Illustrate and describe the set $\{(x, y) \mid x^2 + y^2 = 1\}$.

7 P is at $(-5, 9)$, Q is at $(1, 2)$, and R is on the x-axis. Given that triangle PQR is isosceles, find the possible coordinates of R.



EXERCISE 6A

1 **a** $2\sqrt{2}$ units **b** 7 units **c** $2\sqrt{5}$ units **d** $3\sqrt{5}$ units

e 7 units **f** $\sqrt{5}$ units **g** $\sqrt{10}$ units **h** $\frac{\sqrt{5}}{2}$ units

2 **a** $10\sqrt{2} \approx 14.1$ km **b** $20\sqrt{5} \approx 44.7$ km

c $10\sqrt{26} \approx 51.0$ km

3 **a** isosceles with AB = AC = $\sqrt{85}$ units **b** scalene

c isosceles (and right angled at B) with AB = BC

d isosceles with BC = AC = $\sqrt{7}$ units

e equilateral, all sides $2\sqrt{3}$ units

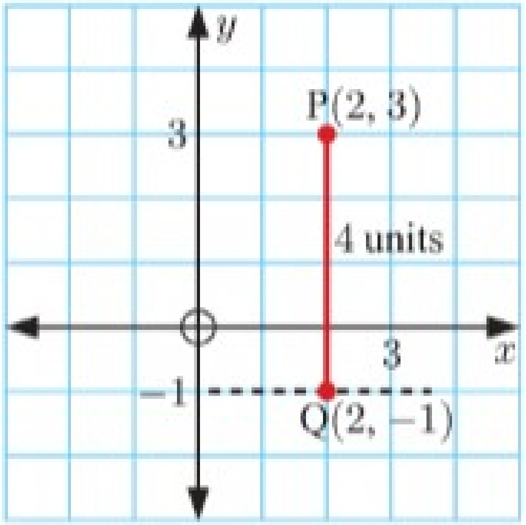
f isosceles (AC = BC) if $b \neq 2 \pm a\sqrt{3}$

equilateral if $b = 2 \pm a\sqrt{3}$

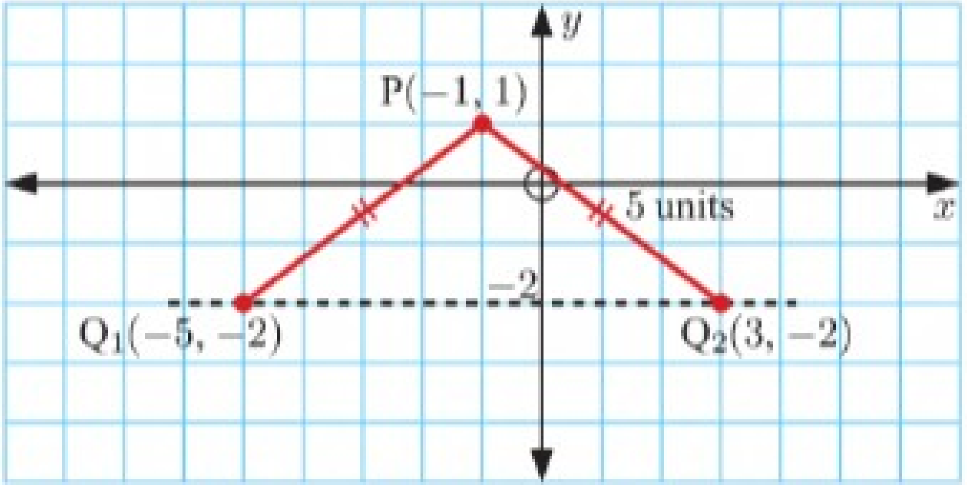
4 **a** right angled at B **b** not right angled

c right angled at A **d** not right angled

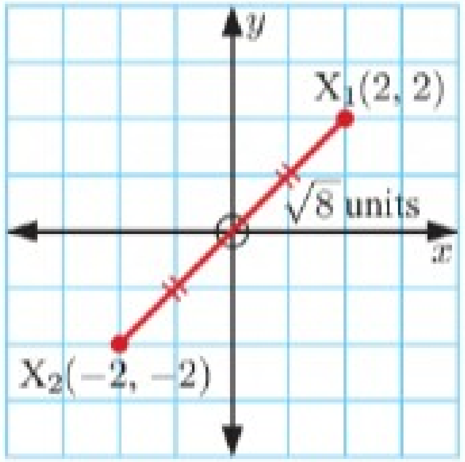
5 **a** $a = 2$



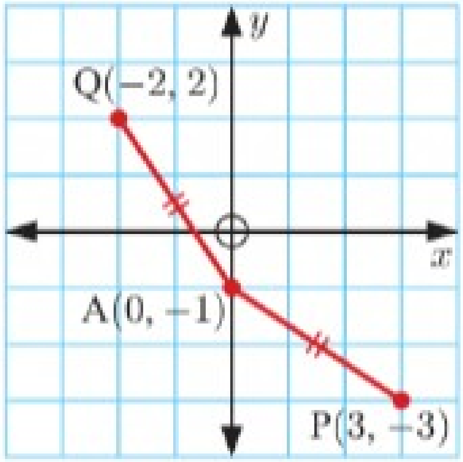
b $a = 3$ or -5



c $a = \pm 2$

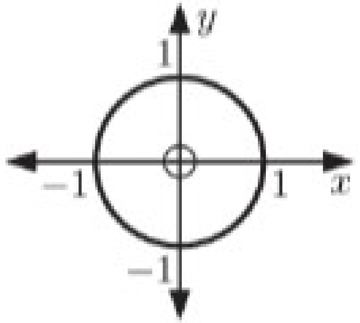


d $a = -1$



6 **a** **i** $x^2 + y^2 = 9$ **ii** $(x-1)^2 + (y-3)^2 = 4$

b



This set represents a circle with centre (0, 0) and radius 1 unit.

7 Let R be $(a, 0)$. Find PQ, PR, and RQ.

Solve PQ = PR, PQ = RQ, PR = RQ.

$(10, 0)$, $(-8, 0)$, $(-3, 0)$, $(-7, 0)$, and $(-\frac{101}{12}, 0)$